# Fundamental Physics Basis for Transport Barriers without Velocity Shear

Understanding Transport Barriers as thermodynamic phenomenon that can be explained by fundamental statistical mechanical concepts

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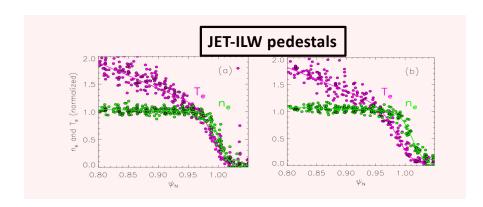
## Powerful ITG/TEM must become weak to have a Transport Barrrier (TB)

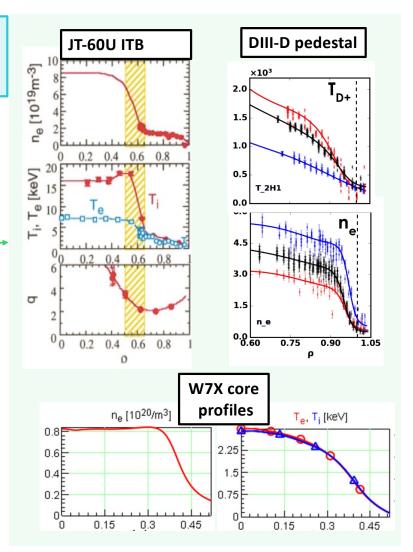
- There exist regimes where this occurs without velocity shear
- A new Analysis within the framework of Constrained statistical mechanics shows the WHY?

TB's can arise because a fundamental constraint prevents the dynamics from accessing the enormous gradient free energy

- New theoretical concepts demonstrated by extensive gyrokinetic simulations
- Some present experiments are in this regime, future ones likely have to be

The constrained Stat-Mech approach reveals connections to basic physical dynamics, and "universal" behaviors, that are not evident in the usual dispersion relations





TBs are a HUGE puzzle from a general statistical mechanics perspective (many degree of freedom systems like gyrokinetics)

Turbulent transport in TBs BEHAVES EXACTLY THE OPPOSITE to a TRULY VAST VARIETY of physical systems outside of magnetic confinement, where:

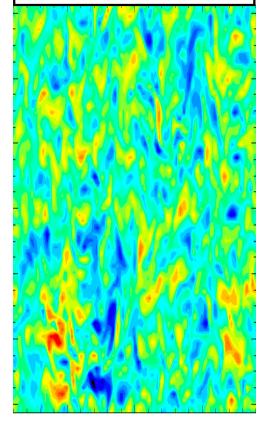
Increasing some gradient (a "thermodynamic force")
Increases thermodynamic relaxation rate (the corresonding "thermodynamic flux")

In TBs, higher gradients DO NOT lead to higher turbulent fluxesoften, in fact, the opposite

Given the **truly huge** weight of physical systems that behave the "usual" way, we are compelled to ask:

What *exceptional* process operates in TBs in magnetically confined plasmas? (Specifically, when velocity shear is not crucial.)

2D contour plot of TB gyrokinetic simulation  $\delta n$  (the full 5D phase space is far more complex with far more degrees of freedom)

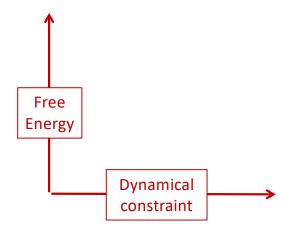


## The answer: the properties of the ITG/TEM arise from the simultaneous action of <u>TWO</u> fundamental dynamics

- 1) Classic free energy considerations
  - Just as in a VAST variety physical systems, as mentioned above
- 2) A dynamical constraint specific to magnetized plasmas:

All gyrokinetic fluctuations cannot lead to a net charge flux (to lowest order in the gyrokinetic expansion, transport is "intrinsically ambiploar")

• These two dynamics arise from VERY different physics (their physical origins are "orthogonal", so to speak)



In a bit more detail:

## Classic free energy considerations

- THIS IS **VERY GENERAL**, BASIC, STATISTICAL MECHANICS
- Instabilities are a way to *literally transfer* the free energy in equilibrium gradients into their fluctuations, causing them to grow
- Gyrokinetics has a so-called "entropy" or "free energy" equation to describe this (discussed by many, most recently Helander PRL 2021)

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Schematic of the (somewhat complicated) free energy equation for gyrokinetics

$$\frac{d}{dt} \left( \begin{array}{c} \textit{fluctuation} \\ \textit{free energy} \end{array} \right) =$$

$$-\frac{d}{dt} \left( \begin{array}{c} \textit{equilibrium} \\ \textit{free energy} \end{array} \right)$$

$$-\left( \begin{array}{c} \textit{collisional} \\ \textit{entopy production} \\ \textit{Boltzmann H theorem} \end{array} \right)$$

where: 
$$-\frac{d}{dt} \left( \begin{array}{c} equilibrium \\ free\ energy \end{array} \right) \sim \sum_{species} \left( Q_s \frac{dT_s}{dx} + \Gamma_s \frac{dn_s}{dx} T_s \right)$$
thermodynamic fluxes that relax
their respective thermodynamic gradients:
flux
flux

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- Gyrokinetics has a so-called "entropy" or "free energy" equation to describe this (discussed by many, most recently Helander PRL 2021)
- Example: this is the basis of critical dT/dx of ITG- the gradient must be strong enough to transfer sufficient free energy to fluctuations to enable growth
- From this fundamental equation, it would seem: stronger gradients "should" produce higher instability growth ratesthe "usual" behavior
- BUT a basic constraint from magnetized plasma physics can completely prevent the "usual behavior"....

Schematic of the (somewhat complicated) free energy balance equation for gyrokinetics

$$\frac{d}{dt} \left( \begin{array}{c} \text{fluctuation} \\ \text{free energy} \end{array} \right) =$$

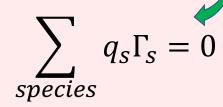
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thermodynamic fluxes that relax Heat Particle their respective thermodynamic gradients: flux flux

### The dynamical constraint that must be satisfied for all gyrokinetic fluctuations: I

- Gyrokinetic transport cannot lead to a net charge fluxit's "intrinsically ambipolar"
  - Well known to be true to lowest order in gyrokinetic expansion (Waltz 1982, Kotschenreuther 1982, Helander 2008, Parra 2012, Parra and Catto 2013, Catto 2019, Kotschenreuther 2021)
  - We'll call this constraint the "Flux Constraint (FC)"
  - The physics behind this constraint is described by various authors as local frame invariance or local momentum conservation in a strongly magnetized plasma
  - NOTE that this is TOTALLY different from the physical origin of free energy considerations



where:

 $\Gamma_s$  is the charge flux from gyrokinetic fluctuations

 $q_s$  is the charge

### How does the Flux Constraint (FC) prevent strong ITG/TEM?

## If the flux constraint is not soluble for $\gamma>0$ , *NO instability is possible*

(and we'll find nonlinear results follow the linear)

#### AND:

Analytic bounds on the flux constraint are ENORMOUSLY simpler to obtain than computing the full dynamics

We'll obtain such bounds, and compare to simulations: it will be obvious that the FC can control the "thermodynamic relaxation" -heat flux, instability growth rate

Striking, "universal" behaviors are found in the simulations that are manifestations of the FC

Of practical importance: the FC controls when weak instbailities are present despite steep gradients (e.g. TBs)

## This is a *qualitatively different WAY of understanding instabilites* (and transport) than the usual dispersion relation approach

#### A dispersion relation combines all the physics

Here we are conciously trying to separate out the consequences from one particular aspect of the physics – the Flux Constraint

From a conceptual point of view:

Dispersion relations obviously do show modes can become stable

Dispersion relations don't easily reveal when stability is a consequence of one particular part of the dynamics e.g., solubility of the FC

#### E.g., ITG dispersion relations have been around for about 60 years without this being realized

In the dispesion relation approach, stability appears as a algebraic consequence, without a simple answer to:

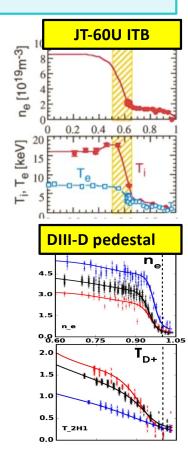
WHY CAN'T THIS PHYSICAL SYSTEM WITH VERY MANY DEGREES OF FREEDOM FIND SOME WAY TO ACCESS STRONG FREE ENERGY LIKE MOST OTHER PHYSICAL SYSTEMS WITH MANY DEGREES OF FREEDOM?

Here an answer is provided: there is NO growing fluctuation which satisfies a necessary and basic dynamical constraint, the FC, so free energy becomes inaccessible to the system

To obtain analytic results to compare to simulations, we use a simplification akin to one often used in statistical mechanics: mean field theory

- Mean field theory: the action on many similar degrees of freedom are treated as a single average quantity
- Our version: the Simplified Kinetic Model that replaces eigenfunctions with a few "eigenfunction averaged" quantities
  - Specifically:  $\langle k_{\parallel} \rangle$ ,  $\langle k_{\perp} \rangle$ ,  $\langle \omega_d \rangle$ , etc.
  - In a TB-like limit of steep gradients, expressions for the averages are derivable
  - SKiM was presented at a previous PPPL talk in Dec 2019
- Even with these simplifications, the disperion relation for  $\omega$ ,  $\gamma$  is far too complicated to solve analytically in general

But simple analytic results for the solubility of the FC can be obtained in important cases, checked against simulations, and they follow them!



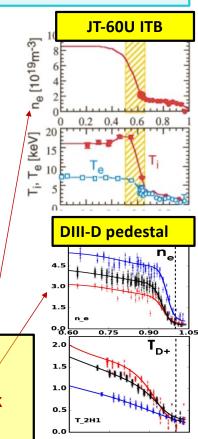
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One result: Often the FC cannot be satisfied for a large enough density gradient

Such density gradients are often present in TBs and are crucial to creating weak ITG/TEM that make the TB possible



## First analytic result: for the ITG with adiabatic electrons ( $ITG_{ae}$ )

Define a normalized density gradient:

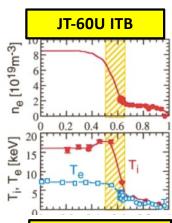
$$F_p = \frac{T \ dn/dx}{d \ (nT)/dx} = \frac{Fraction \ of \ the \ density \ gradient}{In \ the \ pressure \ gradient}$$

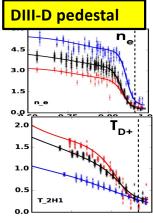
(related to the usual  $\eta_i$ , but  $F_p$  will be more convenient)

The flux constraint for the ITG<sub>ae</sub> cannot be satisfied if:  $|F_p \ge 0.6$ 

Contrast the simplicity of this criterion to the complexity of a dispersion relation that can't be solved analytically!!

> The profiles of many TBs satisfy such conditions from the FC: these bounds are experimentally relevant





Testing the analytic result with simulations in realistic geometry for ITG with adiabatic electrons (  $\overline{ITG}_{ae}$ )

Recall: flux constraint for the ITG<sub>ae</sub> cannot be satisfied if:  $F_p \ge 0.6$ 

Simulation procedure: start with linear instabilites

For various TB-like geometries and parameters, FIX a large ion Temperture gradient ( $R/L_T \sim 10-80$ ) and increase density gradient ( $R/L_n$ )

- Well known: with adiabatic electrons, density gradients can't drive an instability, because the density gradient can't be relaxed
  - This is an example of a **free energetic argument**: free energy in density gradients is not accessible to instabilites with adiabatic electrons

BUT the constant dT/dx is **always** a strong free energy source in these simulations as dn/dx increases

There is no **free energetic** reason why the large temperature gradient can't drive instabilitesquit the contrary, strong instabilites would be expected to be present based upon typical behavior in other physical systems Simulation results for ITG<sub>ae</sub> demonstrate fundamental physics points (and are VERY surprising according to previous expectations)

• Previous expectation:

If some factor **strongly** affects the ITG growth rate, it should affect the stability point for density gradients (the well known critcal  $\eta_i$  or critical  $F_p$ )

Simulations find that this is almost completely untrue

On the contrary, simulations show that it is important to distinguish between energetic effects and the solubility of the flux constraint

• Simulations results are the concrete manifestation of a basic physical fact whose importance was unrecognized before now:

The free energy equation is strongly affected by many factors → that have no effect upon the upper bound for solubility of the flux constraint

By inspection, for the ITG<sub>ae</sub> the respective equations are affected by these variables

#### The free energy equation:

The *absolute magnitude* of the gradient ( $R/L_T$ )

The magnetic geometry

The ratio  $T_i/T_e$ 

Impurity content (~dilution of D)

## The flux constraint bounds: NONE OF THE THINGS ABOVE

**Ratios** of the gradients  $(F_n \text{ or } \eta)$ 

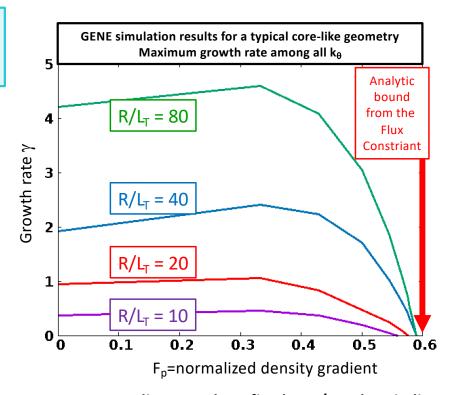
Impurity content (*more* strongly than the mere dilution effect above)

Huge variations in the T gradient strongly affect the free energy equation, and hence the growth rate, but not the stability point in  $F_p$ 

- Simulate temperature gradients R/T<sub>T</sub> far above marginal stability (3x to 25x)
- Increase the density gradient until stability
- Simulations: the critical F<sub>p</sub> is always close to the analytic upper bound set by the flux constraint

The growth rate varies by ~ 10 x

But the critical  $F_p$  varies by  $\sim 5\%$ 



Temperature gradients are kept fixed at R/L<sub>T</sub> values indicated Density gradient is increased until stability

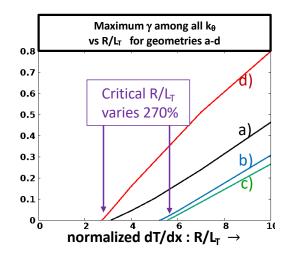
- 1) Huge changes in the free energy drive have little effect on the FC solubility
- 2) Systems with many degrees of freedon "find a way" to approach the "hard" FC limit rather closely BUT DON'T GO BEYOND THE SOLUBILITY LIMIT

Huge variations in magnetic geometry don't change the crtitical  $F_p$ , even though they greatly affect the critical dT/dr

## Geometry strongly affects critical dT/dx via the free energy equation

- We simulate:
  - a) typical H-mode in the core
  - case with highly negative shear and Shafranov shift
  - c) Case with MUCH larger Shafranov shift
  - d) highly unstable L-mode

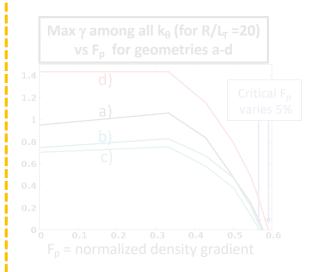
#### $\sim$ 270% variation in normalized dT/dx = R/L<sub>T crit</sub>



#### The critical F<sub>n</sub> varies only ~ 5%

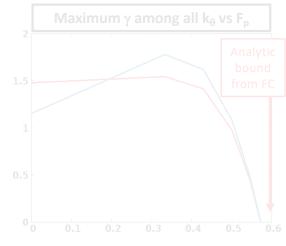
It is always very close to the analytical bound from the flux constraint

The critical  $F_p$  is clearly determined by the Flux Constraint



## The same phyiscs considerations apply to stellarator geometries

- W7X (Max J)
- NCSX (Quasi sym)

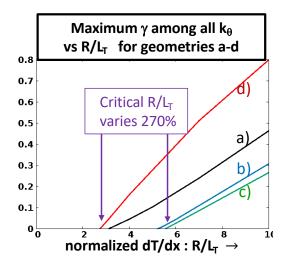


 $F_p$  = normalized density gradient

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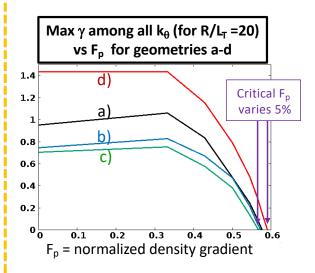


#### Increasing the density gradient:

The critical F<sub>p</sub> varies only ~ 5%

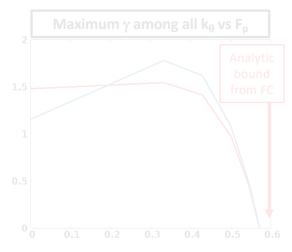
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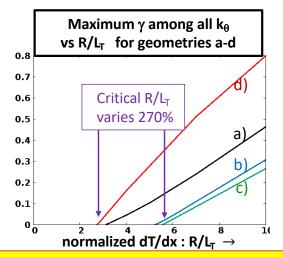


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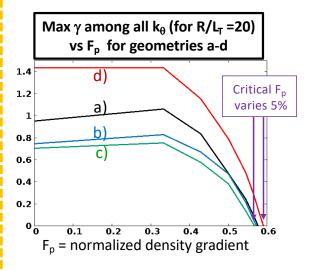
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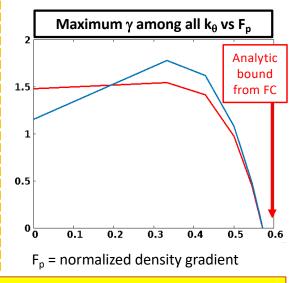
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The critical  $\boldsymbol{F}_p$  is clearly determined by the Flux Constraint



## The same phyiscs considerations apply to <u>stellarator</u> geometries

- W7X (Max J)
- NCSX (Quasi sym)



Again: factors that strongly influence the free energy dynamics don't change the consequence of the Flux Constraint bound

We have considered other large variations in parameters: previous behavior is found to be "universal"

#### Vary $T_i/T_e$ from $\frac{1}{4}$ to 4

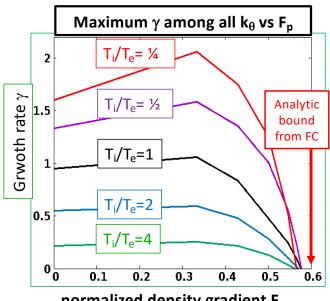
- This changes critical dT/dx by ~ 400%, beacuse:
- T<sub>i</sub>/T<sub>e</sub> has a strong effect upon the free energy equation
  - It changes the relaitve size of the energetically stabilizing adiabatic electron term ~e\psi/T<sub>e</sub>
- But adiabatic electrons have no charge flux

so the FC is unaffected by T<sub>i</sub>/T<sub>e</sub> Ans so, the  $F_n$  bound is

unaffected by T<sub>i</sub>/Te

And what do simulations say?

The critical  $F_p$  is always close to the analytical FC bound (varies ~4%)



normalized density gradient Fp

We have considered other large variations in parameters: previous behavior is found to be "universal"

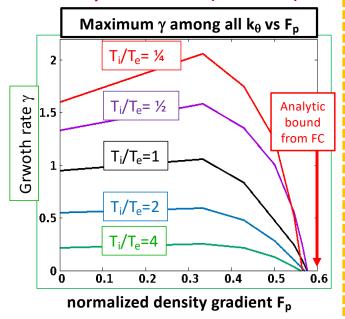
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- But adiabatic electrons have no charge flux

so the FC is unaffected by  $T_i/T_e$ 

Varying the density gradient:

The critical  $F_p$  is always close to the analytical FC bound (varies ~4%)



- All cases we have examined with steep gradients show similar behavior
  - Many diverse geometries and parameters
- The insolubility of the flux constraint is the underlying reason for stabilization of the ITG<sub>ae</sub> by density gradients
- 2) This introduces a remarkable universality of behavior in extremely diverse cases, largely regardless of the free energy considerations
- 3) The behavior of the gyrokinietc system is reminiscent to other physical systems with many degrees of freedom:

The system "finds a way" to access free energy unless a "hard constraint" prevents it- here, the flux constraint

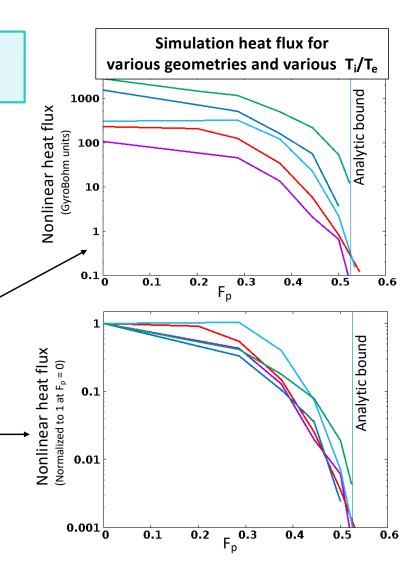
and

if so, it "finds a way" to approach the boundary of the hard constraint rather closely The flux constraint is more restictive for low k modes- which are the most relevant ones for NONLINEAR heat fluxes

- In the cases above, the instabilites near F  $_{\rm p}$   $^{\sim}$  0.6 all had:  $k \rho_i \gtrsim 1$
- BUT: strong nonlinear heat fluxes are dominated by modes  $\,k
  ho_i\,\ll 1\,$
- Nonlinear results follow a more restrictive criterion for low k modes, which we'll derive next
- So nonlinear simulations also have a "universal" behavior, like the previous linear results- but the " $F_p$  bound" is different from 0.6

The analytic flux constraint for low k modes is more relevant for NONLINEAR heat fluxes

- For low k modes the approximte analytic bound is  $F_p \sim 0.53$ 
  - And this low k bound depends on parameters such as impurity fraction, and their gradients
- A wide range of nonlinear cases follow this (different geometries, different gradients, different T<sub>i</sub>/T<sub>e</sub>) (Here Z<sub>eff</sub>=1)
- This is especially apparent if the results are normalized to unity at F<sub>p</sub> =0



### Analytical result: solubility condition for low k modes (An expansion of full equations in k)

- Solubility condition for F $_{
  m p}$  now depends on  $m{k}_{\perp}m{
  ho}_{m{i}}$ 
  - · Arises from gyro-averaging effects
- Instability can reach higher  $F_p$  for higher  $k_\perp \rho_i$

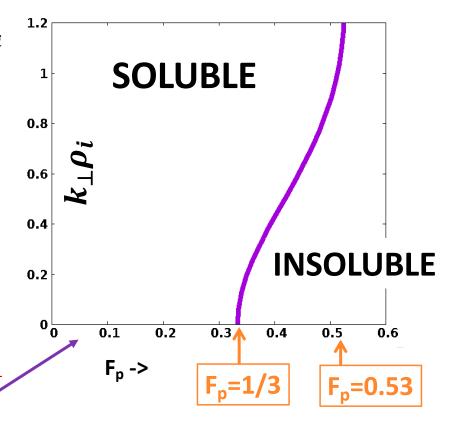
The low k modes that dominate the heat flux go stable for  $^1/_3 \le F_p \lesssim 0.53$ 

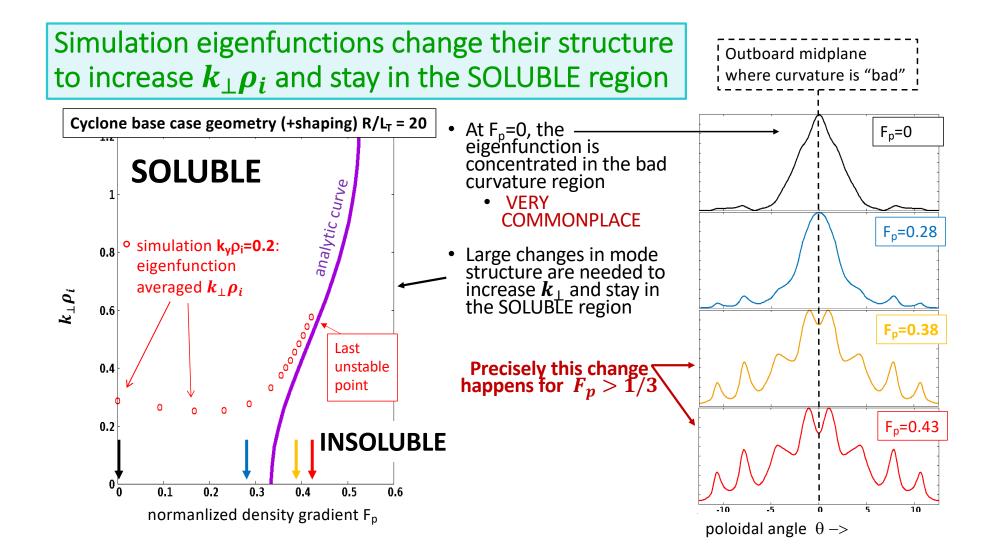
(Many experimental TBs have F<sub>p</sub> in roughly this range)

 Remarkable demonstration that the FC is controlling simulations:

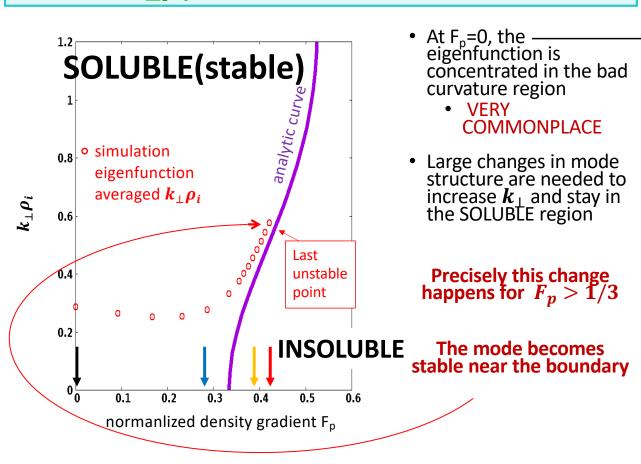
linear eigenmode shape changes to stay in the soluble region as F<sub>p</sub> is increased.....

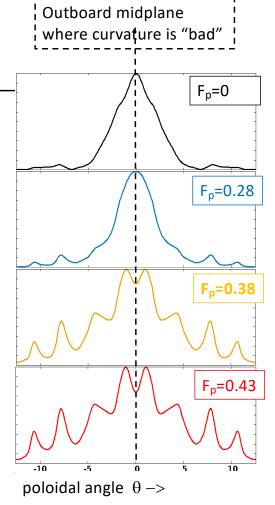
The role of  $k_{\perp}$  is played by an *eigenfunction average* of  $k_{\perp}$ Both linear simulations follow this curve





## The eigenfunctions change their structure to increase $k_{\perp}\rho_i$ and stay in the SOLUBLE region

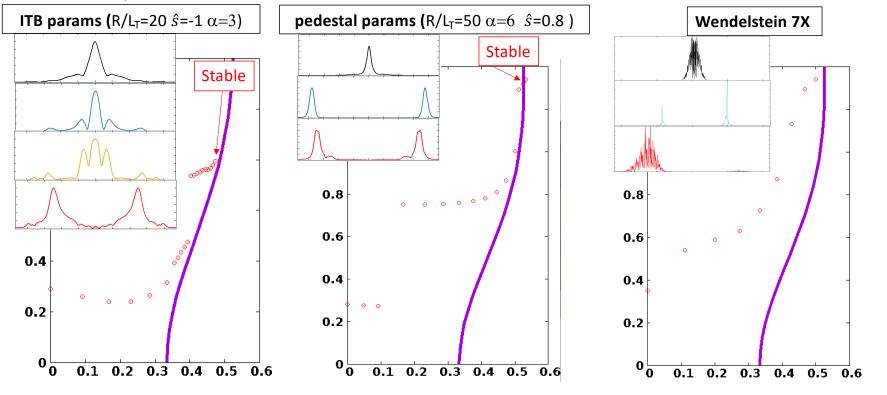




## Such changes happen generically for different geometry and R/L<sub>T</sub>

low k modes always have considerable changes to stay above the analytic FC curve when  $F_{\!p}>1/3$ 

• Shown for k<sub>y</sub>p<sub>i</sub>=0.2: Details of the eignefunction evolution differ on a case by case basis, but the solubility curve is respected

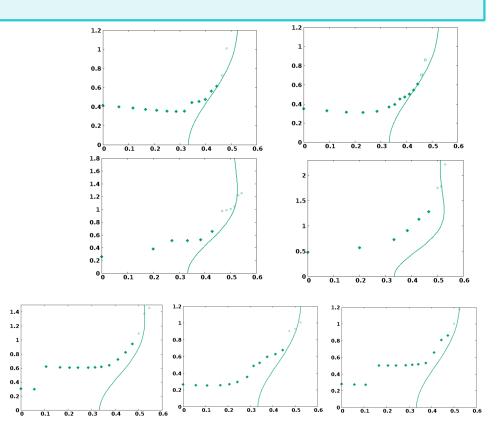


### A tremendous number of geometries have been tested, and show this behavior

- The generic bahavior is for the eigenfunction to change and increase the average  $k_\perp \rho_i$  as  $F_p$  is increased
- Not realized previously (to our knowledge)
- Plots show that the increase follows the analytic curve derived from the FC

This is striking evidence that the FC is controlling the eigenfuncion

as F<sub>p</sub> approaches the analytic limit



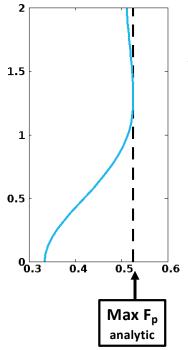
## 

## Nonlinear ITG<sub>ae</sub> follow this boundary too

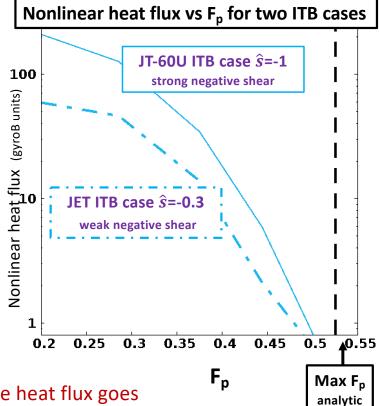
- Define: max value of  $F_p$  for all  $k_\perp \rho_i$  = a stability boundary for all low k modes
- We would expect that nonlinear heat fluxes become small as this value is approached since
  - $k_{\perp}$ becomes large, so eddies are smaller
  - $k_{\perp}$  becomes large, so nonlinear coupling to stable modes becomes stronger
  - eventually the mode becomes stable

Simulations for diverse geometries and gradients show that the heat flux goes down by  $\sim$  2 orders of magnitude as the analytic  $F_p$  boundary in approached......

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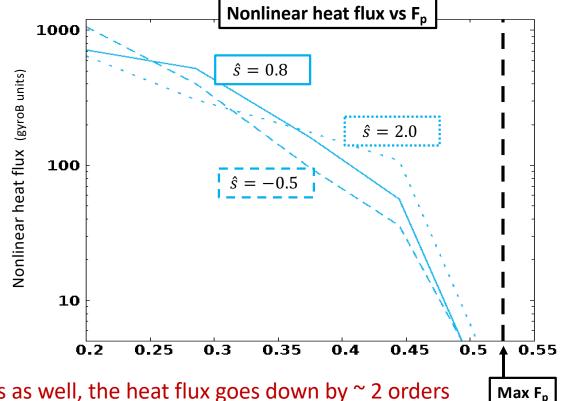
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## Nonlinear runs for diverse cases: pedestal paramters too

- All pedestals have high  $\alpha$  & large R/L
- But magnetic shear of pedestals varies a lot
- Three pedestal like cases:
  - $\hat{s} = -0.5, 0.8 \ and \ 2$
  - $\alpha = 6$  ,  $R/L_T = 50$



analytic

For pedestal-like geometries as well, the heat flux goes down by  $^{\sim}$  2 orders of magnitude as the analytic  $F_p$  boundary in approached

### Impurity (Z) charge fluxes make the FC more restrictive (NOT a dilution effect)

• Z and their gradients are often significant in TBs:

Impurities typically contribute little to the free energy (mainly diluting deuterium free energy),

But due to their higher Z, they contribute disproportionately to the total charge flux

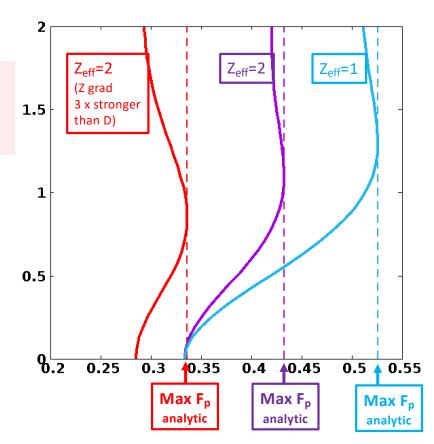
The higher Z tendency to produce larger charge flux implies: density gradients must be lower to satisfy the FC

Stronger impurity gradients than deuterium gradients accentuate such effects

Stronger impurity gradients than deuterium or electrons are often found experimentally in TBs

#### Impurities lower the analytic max F<sub>p</sub>

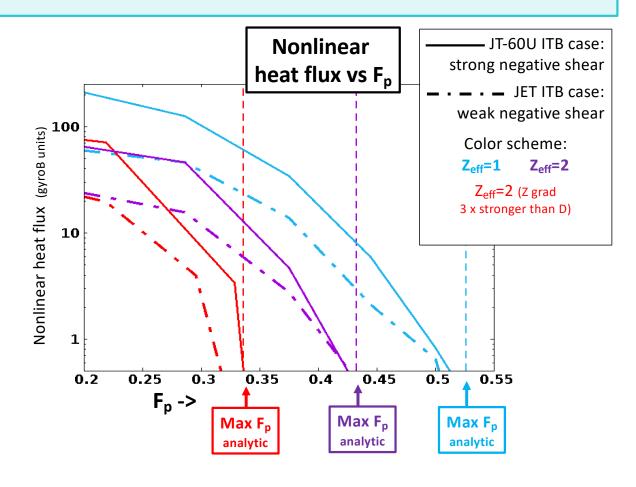
Simulations follow the Z modified solubility boundary like the previous results with  $Z_{\text{eff}}$ =1



## Nonlinear runs for diverse geometries follow the constraint boundary with Z: ITBs

- For the previous ITB geometries:
- the heat flux goes down by ~2 orders of magnitude as the analytic F<sub>p</sub> boundary in approached, and

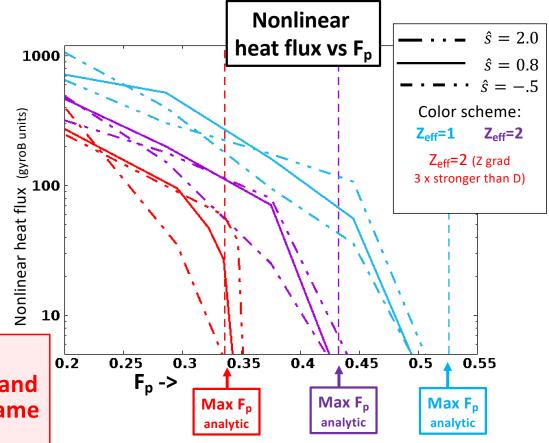
NONLINEAR heat flux follows the impurity modified FC solubility condition



### Nonlinear runs for diverse geometries follow the constraint boundary with Z: pedestals

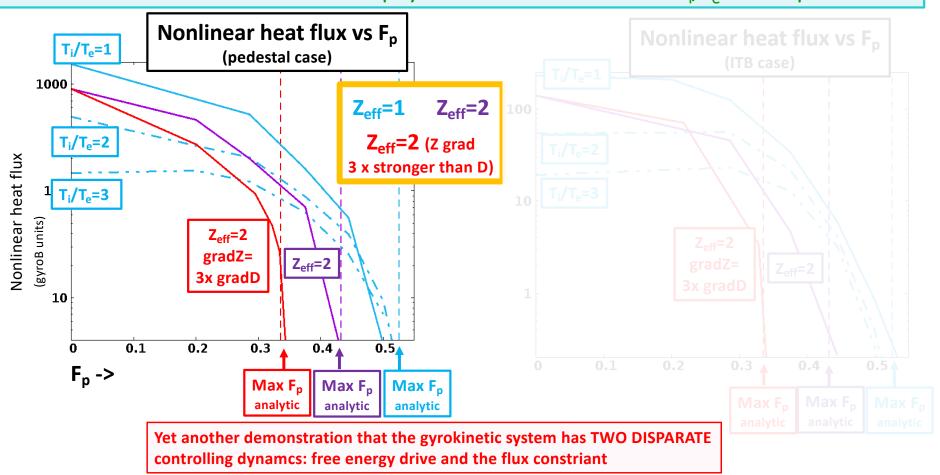
- For three different pedestal geometries
- the heat flux goes down by ~2 orders of magnitude as the analytic F<sub>p</sub> boundary in approached

NONLINEAR heat flux follows the impurity modified FC solubility condition

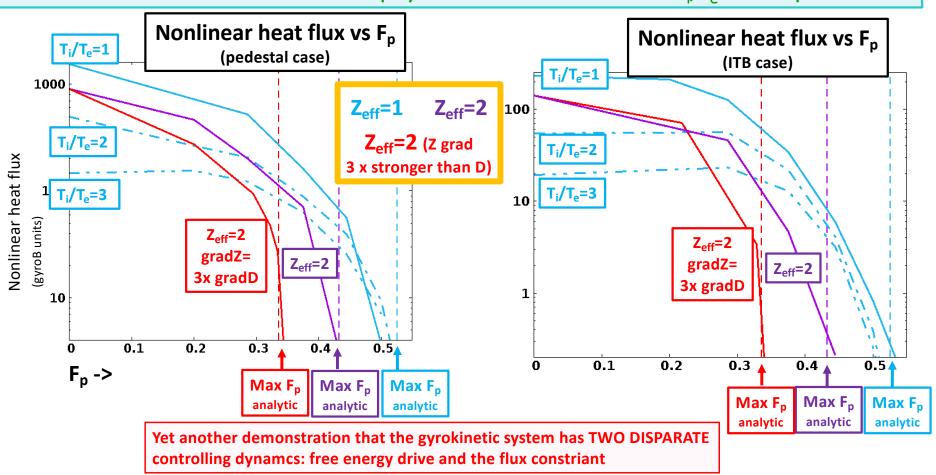


Despite HUGE parameter differences between pedestal and ITB cases, they all follow the same analytic constraint bounds

The  $T_i/T_e$  can severely alter the energetics, but not the FC solubility. The nonlinear simulations reflect this fundamental physical difference between  $T_i/T_e$  and impurities

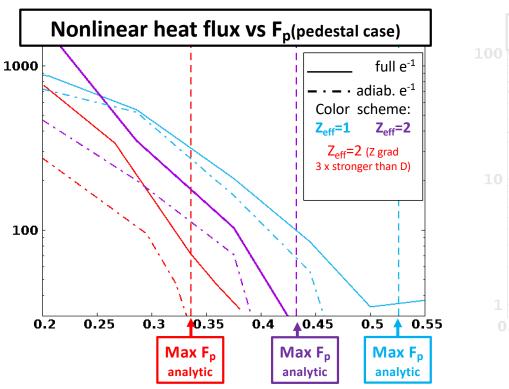


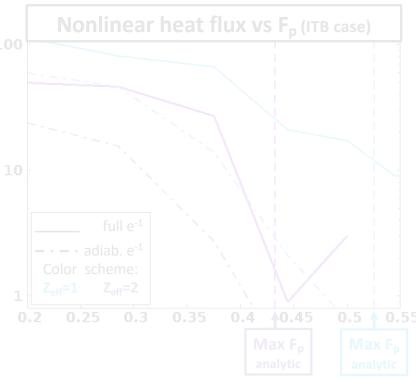
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## Simulations with full electrons and electromagnetic effects for TB cases behave similarly to $ITG_{ae}$ , but the electrons reduce the stabilization

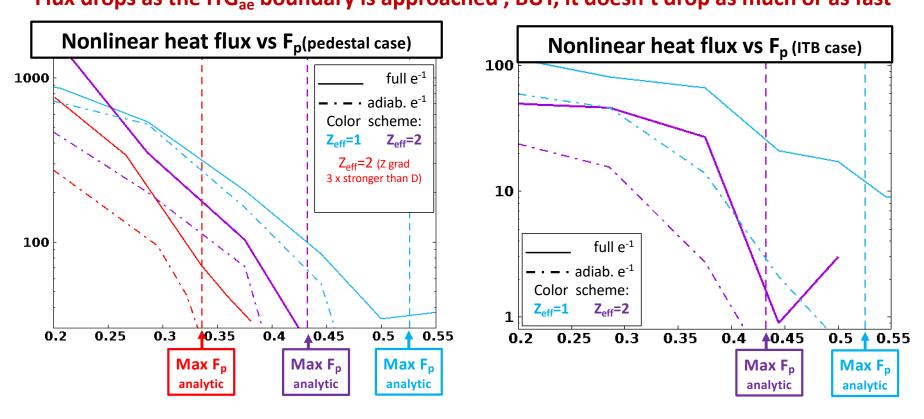
• The electrostatic part of the heat flux behaves similarly to the ITG<sub>ae</sub> for both pedestal and ITB cases: Flux drops as the ITG<sub>ae</sub> boundary is approached, BUT, it doesn't drop as much or as fast





## Simulations with full electrons and electromagnetic effects for TB cases behave similarly to $ITG_{ae}$ , but the electrons reduce the stabilization

• The electrostatic part of the heat flux behaves similarly to the ITG<sub>ae</sub> for both pedestal and ITB cases: Flux drops as the ITG<sub>ae</sub> boundary is approached, BUT, it doesn't drop as much or as fast



#### Analysis of TB cases with full electrons finds: again, the flux constraint controls stability

- Again, this is a new way to understand instabilities in TBs
- Conventional understanding of this regime has focused on differences in the properties of ITG and TEM modes
- In actual TBs, however, the fluctuations have major components of each of these simultaneously

## The present analysis is not in terms of two mode types (ITG or TEM), but in terms of two fundamental equations

- The Free-Energy balance equation and the charge Flux Constraint
- These equations apply to ALL fluctuations
- It is not necessary to (futiley) attempt to distinguish whether a fluctuation is primarily an ITG or a TEM
- Rather, one only has to determine when the solubility of the flux constraint prevents an instability

#### Summary of extensive analysis of TB cases with full electrons:

- Based upon a LARGE amount of simulations and analysis:
  - In the TB regime, the electrons are destabilizing because they contribute a charge flux that makes the FC *easier* to satisfy

Electron charge flux has the opposite effect to the impurity charge flux because the electron charge is negative; so typical electron fluxes make it easier to satisfy the constriant, impurities make it harder

• Electrons reduce the stabilization from the FC=> larger F<sub>n</sub> is required for stability, or, stability may not result

When strong T<sub>i</sub> gradients are present, stability of coupled ITG/TEM requires that the electron charge flux is low, and this requires that the electrons have, primarily, a Boltzmann response (adiabatic)

Phase space averaging, as in statistical mechanics, leads to such an electron response,

Phase space averaging, as in statistical mechanics, leads to such an electron response including for the trapped particles

- For tokamak TBs, this typically requires sufficiently low or negative magnetic shear, or large enough Shafranov shift
   But the general physics of phase space averaging can arise in other circumstances, e.g., some stellarator geometries
- Previous explanations of regimes like this have been interpreted as the energetically stabilizing effects of orbit average "good" curvature for trapped electrons

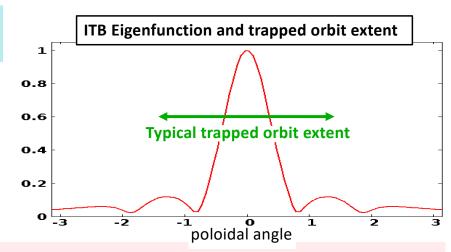
However, just as for the ITG<sub>ae</sub>: energetic effects are not the dominant determant of stability with increasing F<sub>p</sub>

The flux constriant is the controlling physics even with electron dynamics

## Approximate electron "adiabaticity": expected from basic statistical mechanics...

- "Adiabatic", in the ITG/TEM context, means a Boltzmann distribution  $\sim e^{-(\frac{1}{2}\,m\,v^2-q\phi)/T}$
- Basic statistical mechanics: distribution functions become Boltzmann when particles have time to sample all of phase space uniformly.

Electrons are very fast and "try" to do this.



• But: trapped electrons are very constrained in the phase region they can sample.

Nonetheless, if the width of the trapped orbit is much wider than the eigenfunction the region sampled becomes <u>large enough</u> so the <u>response approaches Boltzmann</u>

- Simulations show that when the shear becomes more negative, the eigenfunction becomes narrower in the poloidal angle
- Thus, negative shear and/or high Shafronov shift leads to approximate "adiabaticity" by phase space averaging

Approximate electron "adiabaticity" of ITG/TEM with negative shear is a specific instance of the very general and robust dynamics of phase space averaging in statistical mechanics

Such general dynamics leads to TBs in <u>extremely</u> different geometries, such as stellarators (e.g. W7X and NCSX), where the details of the geometry are different but the same fundamental statistical mechanics operates

### We now make 2D plots of free energy balance and flux constraint for ITG/TEM instabilities

- The 2D plot will be the real frequency  $\omega_r$  and growth rate  $\gamma$
- Use the mean field theory (Simplified Kinetic Model SKiM)
- It gives tractable closed form expressions for the energy balance and Flux Constraint as a function of complex w
- We then plot the energy balance curve and flux balance curve in the complex  $\omega$  plane
- The actual eigenvalue  $\omega$  is given by the intersection of the Free energy and FC curves

  This can be shown to be <u>exactly</u> equivalent to the usual dispersion relation

But these plots allow us to see the physical origin of the stabilization by disentangling two different physical effects:

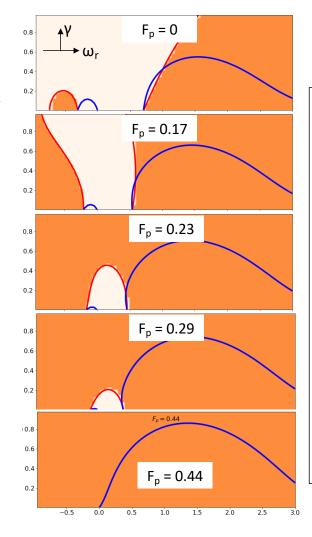
Free energy drive and flux constraint



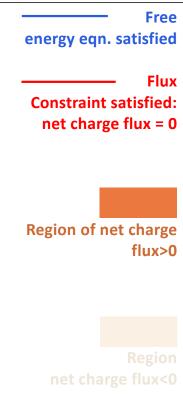
## 1<sup>st</sup> consider a typical case with adiabatic electrons

- When  $F_p$  is small, the Free energy curve primarily limits  $\gamma$
- As density gradients (Fp) increase
  - Free energy curve: little change
  - FC curve: HUGE change
- As F<sub>p</sub> increases:
  - The orange region of positive charge flux expands as the density gradient drives more ion flux
  - The FC solution curve gets compressed by the expanding orange region
- Eventually FC is soluble only near  $\gamma$ =0
- By increasing  $F_p$  even further, particle flux is everywhere positive (for  $\gamma$ >0)

No instability is possible since FC cannot be satisfied

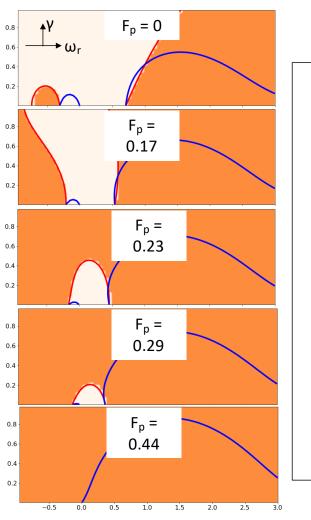




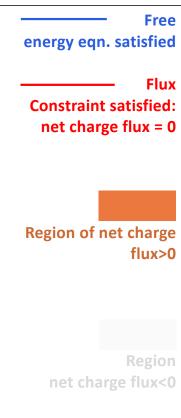


#### Why do the curves evolve this way?

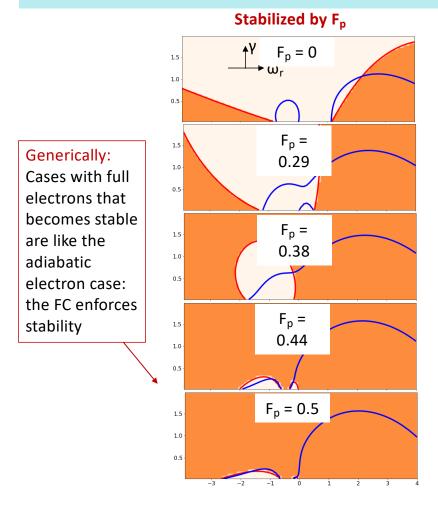
- **Energy curve:** doesn't change qualitatively because there is always a lot of free energy in the equilibrium
- The FC curve:
- As density gradient is increased, the region of outward flux increases
- As expected, density gradient is acting to drive a density flux, and increases it
  - Orange region  $(\Gamma_i > 0)$  grows
  - White region  $(\Gamma_i < 0)$  shrinks
- As density gradient grows, the flux balance curve has a solution only at low  $\boldsymbol{\gamma}$ 
  - A finely tailored velocity response is necessary to give no flux, requiring sharp resonances ( $\gamma$  ~ 0) and a very limited range of frequencies  $\omega$
- Eventually no solution with  $\gamma > 0$  is possible for  $F_p$  large enough the region with  $\gamma > 0$  is all orange

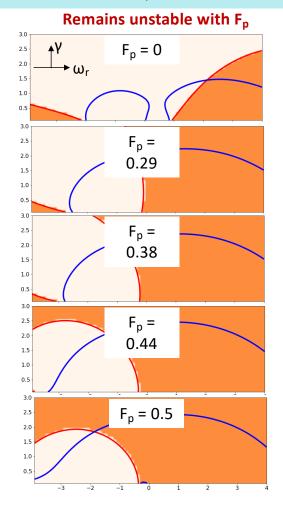


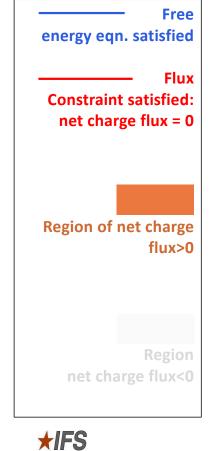
#### \*IFS



### The curves with <u>full electrons</u> for a case with stabilization by F<sub>p</sub> and a case that remains unstable







Statistical mechanical concepts show why the Flux Constraint becomes insoluble at large dn/dx

In TBs, electrons are slightly non-adiabatic, they cause small electrons flux ( $\Gamma_e$ ):

#### **Basic statistical mechanics:**

Thermodynamic strongly tend to drive forces

Their corresponding thermodynamic fluxes

- $\Rightarrow$  Increasing dn/dx increases  $\Gamma_i$  more than  $\Gamma_e$ , due to nearly Boltzmann e<sup>-1</sup> but  $\Gamma_e$  is significant
- $\Longrightarrow$ The small  $\Gamma_e$  makes it easier to balance the FC

 $\Gamma_i + Z\Gamma_Z = \Gamma_e$  (Flux constraint with electrons and impurities)

The FC has a solution until dn/dx becomes so large that  $\Gamma_i$  is too large to be balanced by  $\Gamma_e$ 

Electrons are destabiling while impurity ions are stabilizing because electrons have opposite charge to impurities

A small  $\Gamma_e$  from non-adiabatic electrons extends the unstable range to higher  ${\bf F_p}$  than for the  ${\bf ITG_{ae}}$ 

Simulation trends follow from the FC together with fundamental statistical mechanics

Now lets turn to the details of this....



#### Two sources of non-adiabtic electron response: passing & trapped

- Either are potentially important
- · Each is controlled by different physics
- If either gives enough non-adiabatic response, it can destabilize the mode as F<sub>p</sub> increases

That is: a value of Fp that "would have" stabilized ITGae no longer results in stability

#### 1) Passing electrons

- Passing electrons move rapidly and "try" to average over phase space
  - Then the  $\phi$  from the instability would produces a Boltzman response (  $n \sim e^{e\phi/T}$  ) as in statistical mechanics
    - This is the dynamic response to parallel forces  $E_{\parallel} = \nabla_{\parallel} \varphi$  when "fast" electrons can equilibrate with the perturbation
    - Called "adiabatic" response in gyrokinetics (  $\delta n \sim \delta \phi \; e/T$  )
  - BUT, for the long wavelength part of  $\phi$ , there can be strong departures from this (when  $k_{\parallel} \sim \omega / v_{\parallel e}$ )
  - Fortunately, even at such low  $k_{\parallel}$ , inductive effects can potentially shield out the  $E_{\parallel}$
  - This shielding greatly reduces the non-adiabatic electron response from these scales
  - At typical  $\beta_e$ , the non-adiabatic response from passing electrons is usually less than from trapped electrons

Inductive shielding: *roughly* when magnetic skin depth is shorter than the mode scale

Scales as 
$$\sim (k_\perp \lambda_{skin})^{-1}$$
  $\sim (k_\perp \rho_s)^{-1} \; \beta_e^{1/2} \; (m_i/m_e)^{1/2}$ 

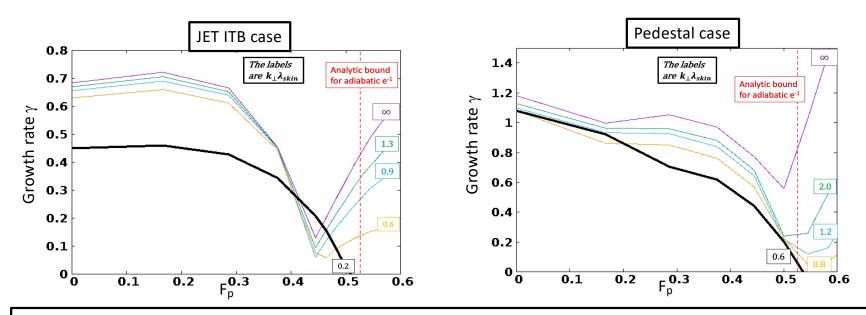
Shielding increases with  $\beta_e/k_\perp^2 \rho_s^2$ 

#### 2) Trapped electrons

- Previous explanations of regimes like this have been interpreted as the energetically stabilizing effects of orbit average "good" curvature for trapped electrons ("drift direction")
- However, as we have seen: energetic effects are not the main determant of stability with increasing F<sub>p</sub>: **the flux constriant is**
- This remains true with trapped electrons
- The most important parameter is not the trapped electron average curvature, but rather, the degree of the trapped electron phase space averaging (just like the passing electrons)

#### First consider only the passing electron response

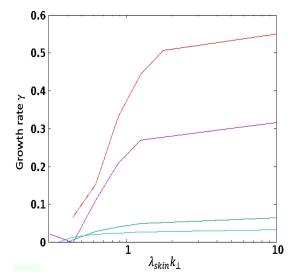
- Perform simulations with neglible trapped particles (aspect ratio  $\epsilon=0$ )
- Consider geometries that are usually expected to produce weak ITG/TEM (negative magnetic shear and/or high  $\alpha$ )
- At fixed geometry parameters (e.g.  $\hat{s}$ ,  $\alpha$ , q), artificially vary  $\beta_e$  (magnetic shielding) from no shielding to the actual value for the equilibrium  $\alpha$



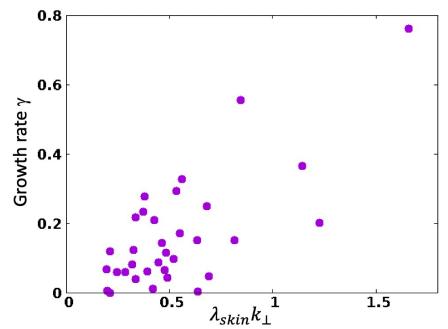
When there is realisitic magnetic shielding, the mode stability is like the ITG<sub>ae</sub>: non-adaibatic passing e<sup>-1</sup> effects are small

### Further simulations with only passing electrons ( $\epsilon=0$ )

- Consider cases with  $F_p=0.5$ , which "would be" stable, except for non-adiabatic response:
  - Growth rates for many different geometries drop strongly as  $\beta_e$  is artificially varied between 0 and the actual value consistent with  $\alpha$



- Consider simulations of large number of geometries where  $\beta_e$  is the actual value
- $\hat{s} \in \{-1 \text{ to } + 1\}, \alpha \in \{0.2 \text{ to } 6\}, q \in \{3 \text{ to } 6\}$



For very diverse geometries:

Low enough  $k_{\perp}\lambda_{skin}$  always leads to low  $\gamma$ 

( $\gamma$  can be low even without skin depth shielding, if the  $k_{\parallel}v_{th\;e}$  is large enough)

Conclusions: passing electrons can be a significant non-adiabatic effect, but:

- Sufficient  $\beta_e$  renders this effect small (typical  $\beta_e$  is enough)
- ullet They can also be small if the  $k_\parallel v_{th\;e}$  is large enough

### Now consider trapped electrons

- These are usually the dominant non-adiabatic electron contributor
- To understand these effects, we begin by adding trapped electrons to the mean field theory (SKiM)

#### The mean field theory with trapped electrons

- The eigenfunction averaged quantities for electrons now involve trapped orbit averages ("bounce averages")
- These expression be derived by the same methodology as for described in earlier presentations (SKiM)

Ion non-adiabatic response

e<sup>-1</sup> non-adiabatic response

$$0 = \frac{1}{T_i} - \frac{1}{T_i} \int dv \ F_M J_0^2 \left( \frac{\langle k_{\perp} \rangle v_{\perp}}{\Omega_i} \right) \frac{\omega - \omega_{*i}}{\omega - \langle k_{\parallel} \rangle v_{\parallel i} - \langle \omega_{di} \rangle} + \frac{1}{T_e} - \frac{\langle f_{Trap} \rangle_{eigen}}{T_e} \int dv \ F_M \frac{\omega - \omega_{*e}}{\omega - \langle \omega_{de} \rangle_{eigen} + i v_{eff}(v)}$$

Eigenfunction averaged coupling to trapped electrons: heuristically, the "effective" trapped fraction.

$$< f_{Trap}>_{eigen} = \\ \frac{\int_{Trapped\ region} dl\ d\Omega_{v} < \phi >_{bounce}^{2}}{\int_{Full\ region} dl\ d\Omega_{v}\ \phi^{2}}$$

This is the over-all strength of the non-adiabatic trapped e<sup>-1</sup> response

This becomes small when  $<\phi>_{bounce}^2$  becomes much smaller than  $\phi^2$ , which often happens for TB parameters

The solid angle  $d\Omega_v$  is in velocity space (pitch angle with magnetic field)

Eigenfunction averaged bounce averaged curvature drift: heuristically, the average curvature for trapped e<sup>-1</sup>

$$= \frac{\int dl \, d\Omega_{v} < \omega_{de} >_{bounce} < \phi >_{bounce}^{2}}{\int dl \, d\Omega_{v} < \phi >_{bounce}^{2}}$$

This gives the energetics of the response: orbit averaged curvature weighted by the strength of coupling for that orbit  $^{\sim} < \phi >_{bounce}^2$ 



#### The conventional physical picture of stabilization of the ITG/TEM is actually not operative

 Conventionally, negative bounce averaged curvature is thought to stabilize the TEM alone, and density gradients stabilize the ITG

HOWEVER, actually:

- Negative curvature stabilizes the TEM alone but NOT the ITG/TEM:
  - The trapped electron response allows the instability to satisfy the FC by creating a  $\Gamma_e$  even if bounce averaged electron curvature is stabilizing

$$q_i \Gamma_i + q_e \Gamma_e = 0$$

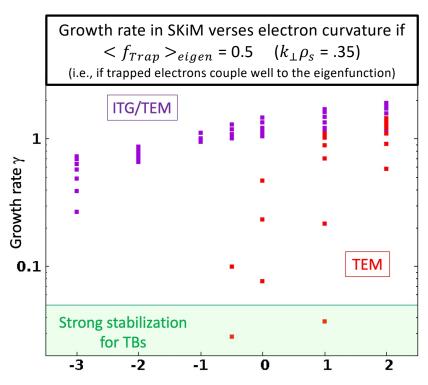
- Creating the requisite  $\Gamma_e$  to allow intability does take some expenditure of free energy
- But for typical parameters: the ions can supply much more that this amount of energy, so  $\gamma$  is only reduced modestly
- So actually, stability results only by reducing the amount of the non-adiabatic electron response (small  $< f_{Trap}>_{eigen}$ ), NOT by making the electron curvature negative ( $<\omega_{de}>_{eigen}$ <0)

### Lets first see this in the generic behavior in SKiM: stabilizing electron curvature stabilizes TEM, but NOT ITG/TEM

- Consider a case with strong coupling to trapped electrons  $< f_{Trap}>_{eigen}$  ~ 0.5
- Also consider F<sub>p</sub>=0.5: would be stable except for trapped electrons
- Vary remaining free parameters of SKiM in a range around typical eigenfunction values

### Lets first see this in the generic behavior in SKiM: stabilizing electron curvature stabilizes TEM, but NOT ITG/TEM

- Consider a case with strong coupling to trapped electrons  $< f_{Trap}>_{eigen} \sim 0.5$
- Also consider F<sub>p</sub>=0.5: would be stable except for trapped electrons
- Vary remaining free parameters of SKiM in a range around typical eigenfunction values
- Making the bounce averaged curvature sufficiently negative stabilizes the TEM
  - The TEM is when we set dT<sub>i</sub>/dx set to zero
- But it does not stabilize the ITG/TEM (including dT<sub>i</sub>/dx), even when electron curvature is FAR more negative than in actual eigenfunctions!



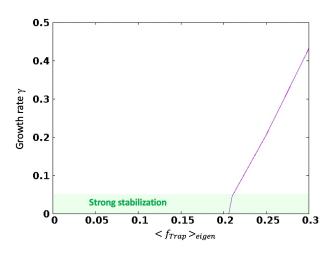
multiplier times typical  $<\omega_{de}>_{eigen}$  found in TB cases

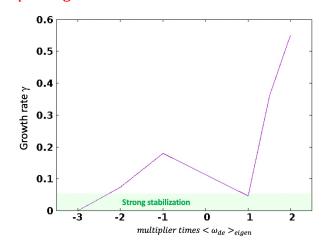
According to SKiM: the trapped electron parameter that gives strong stabilization with  $F_p$  is small  $< f_{Trap}>_{eigen}$  NOT stabilizing electron curvature (when strong  $T_i$  gradients are present)

- Again take  $F_p = 0.5$ : where  $ITG_{ae}$  is stable, so non-adiabatic electrons are necessary for instability
- Evaluate the SKiM coefficients for a typical ITB case, and vary electron parameters to find their sensitivity

According to SKiM: the trapped electron parameter that gives strong stabilization with  $F_p$  is small  $< f_{Trap}>_{eigen}$  NOT stabilizing electron curvature (when strong  $T_i$  gradients are present)

- Again take  $F_p = 0.5$ : where  $ITG_{ae}$  is stable, so non-adiabatic electrons are necessary for instability
- Evaluate the SKiM coefficients for a typical ITB case, and vary electron parameters to find their sensitivity
- Stabilization is achieved by small reductions in  $< f_{Trap}>_{eigen}$  NOT stabilizing curvature





- This characteristic is generic in SKiM.
  - It's also consistent with simulations
  - It's also consistent with the character of the FC

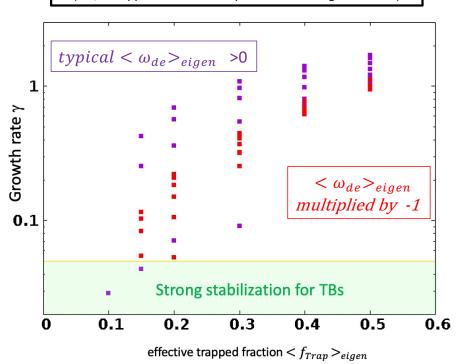
### The magnitude of the non-adiabitic response $< f_{Trap}>_{eigen}$ strongly affects stability

- The growth rates when the electron curvature is stabilizing are indeed smaller, but:
  - The requisite  $< f_{Trap}>_{eigen}$  for the strong stability needed for TBs is not strongly affected by whether the electron curvature  $<\omega_{de}>_{eigen}$  is stabilizing or destabilizing

The Solubility of the FC for strong stabilization is more affected by the magnitude of the trapped electron response  $< f_{Trap} >_{eigen}$  than by the curvature  $< \omega_{de} >_{eigen}$  (which mainly impacts energetic)

As with the ITG<sub>ae</sub> results previously, the solubiity of the FC is controlling stability, not energetic factors

Growth rate verses electron curvature if (i.e., if trapped electrons couple well to the eigenfunction)



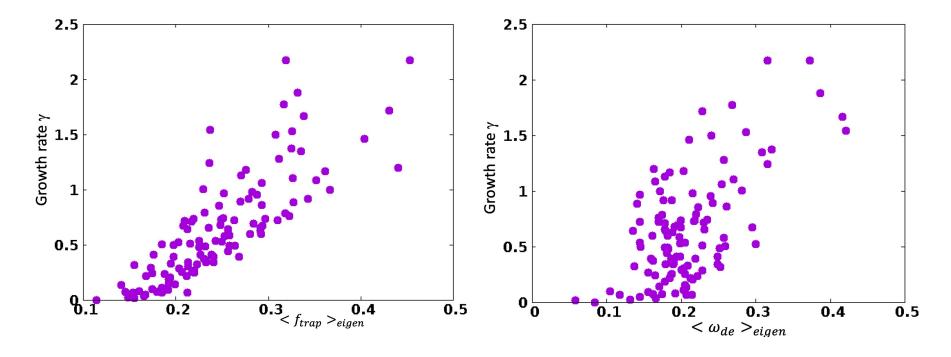
#### Now we consider stability in simulations with full electrons (& electrmagnetic)

- We choose  $F_p$ =0.5 and  $k_\theta \rho_s = 0.35$ , where the mode would be **stable** except for non-adiabatic electrons
- We vary magnetic shear  $\hat{s}$  and  $\alpha$  over a wide range
  - $\hat{s} \in \{-1 \ to \ +1\} \ \alpha \in \{0.2 \ to \ 6\}$
  - This includes values
    - usually associated with TBs
    - · not associated with TBs
    - intermediate values
- We also vary
  - $r/a \in \{0.15 \text{ to } 1\} \text{ for } (R/a=3)$

We evaluate the averages  $< f_{Trap}>_{eigen}$  and  $<\omega_{de}>_{eigen}$  found from simulation eigenfunctions to find out which is associated with strong stabilization

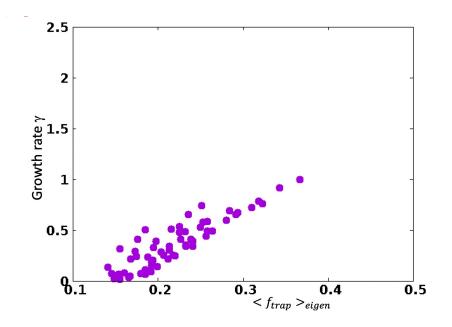
### Simulations results follow the same pattern as SKiM

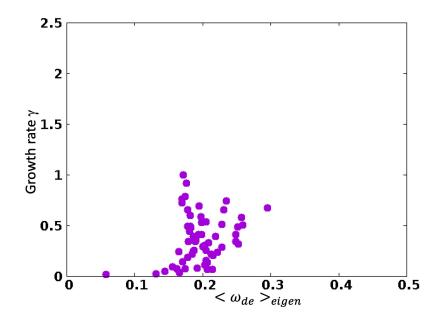
- GENE growth rates: clear association with  $< f_{Trap}>_{eigen}$  and weak association with  $<\omega_{de}>_{eigen}$
- Low  $\gamma$  requires low  $< f_{Trap} >_{eigen}$
- But low  $\gamma$  is not associated with  $<\omega_{de}>_{eigen}$  that is appreciably different from high  $\gamma$



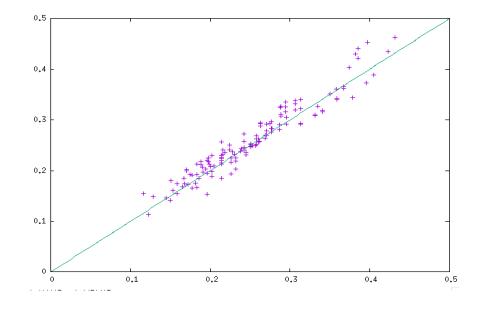
## Strong dependence on $< f_{Trap}>_{eigen}$ becomes clear when we control for the passing electron response

- We can almost remove passing electron effects by considering the subset of cases where  $\lambda_{skin}k_{\perp}$  is small (<0.2)
- Then trapped electrons are the dominant response so the relationship with  $< f_{Trap}>_{eigen}$  becomes clear
- There is still little relationship with  $<\omega_{de}>_{eigen}$





- From the simulations:  $< f_{Trap}>_{eigen} \sim (1+0.085(\hat{s}-\alpha)) \left[\frac{\epsilon}{1+\epsilon}\right]^{1/2}$
- So  $< f_{Trap} >_{eigen}$  is reduced by negative  $\hat{s}$ , high  $\alpha$ . (and small  $\epsilon$ )
- Thus, stability is associated with negative  $\hat{s}$  and high  $\alpha$ , as in experiments and simulations of them
- Also, TBs are more difficult to form at larger major radius, as is also found in experiments





(Exothermic Fusion!)

Future work will be pursued as part of our new company !!!! Mike Kotschenreuther, Swadesh Mahajan, David Hatch, Romi Mahajan

#### **Pursuing Improved confinement sceanarios**

Using density sources and magnetic geometry to create TBs

Novel low recycling divertor design called the *Super-XT* (pat. pending)

Enables exceptional confinement

Low edge n, high edge T

The core can be sustained with high  $F_p =>$  extremely high confinement

Using the Super-XT for extremely high  $\beta$  and confinement in attractive magnetic geometries (pat. pending)

Several patents pending, more to follow very soon!

### **Conclusions**

- We have shown that TB can arise without velocity shear because of a basic constraint in the gyrokinetic system: fluctuation induced transport cannot lead to a net charge flux
- A new conceptual approach to analyze TBs has been developed: based upon constrained statistical mechanics
- By focusing attention on the flux constraint, the conditions that allow TBs can be greatly clarified
- Linear and nonlinear gyrokinetic simulations follow bounds on the solubility of the flux constaint
- As one would expect, various applications follow from this, which are currently under development

In summary: the unique feature of Magnetic Confinement (MC) that can prevent the usual behavior of MANY systems where stronger thermodynamic forces => stronger thermodynamic fluxes

Particle fluxes (a thermodynamic flux) appear in BOTH free energy equation (as is usual) and in the Flux Constraint (unique to MC)

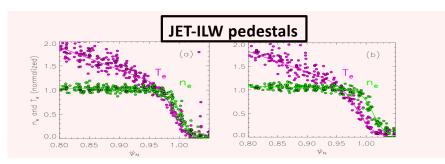
• Unique to MC:  $\Gamma_i + Z\Gamma_Z = \Gamma_e$ 

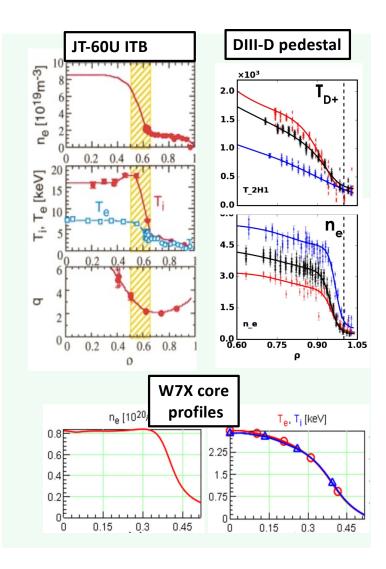
#### THIS CONSTRAINT IS IN TERMS OF THERMODYNAMIC FLUXES: the $\Gamma$

- As usual, the  $\Gamma$  are strongly affected by the corresponding thermodynamic forces: density gradients

Density gradients thus have a DUAL NATURE: as thermdynamic forces that affect BOTH the free energy equation, and ALSO,  $\Gamma$  in the flux constraint

- This DUAL NATURE unique to MC leads to unique consequences: under the "right" circumstances, the thermodynamic forces makes the FC INSOLUBLE for fluctuations.
- This prevents instabilites from arising to access equilibrium free energy, leading to steep gradients without high fluxes: TBs without velocity shear





### How does the Flux Constraint (FC) prevent strong ITG/TEM?

There are conditions where the flux constraint cannot be satisfied for ANY fluctuation with  $\gamma > 0$  (whether or not the fluctuation satisfies Maxwell's equations)

Any instability that satisfies Maxwell's equations must satisfy the flux constriant, so

# If the flux constraint is not soluble for *any* fluctuation, *NO instability is possible*

(and we'll find nonlinear results follow the linear)

Bounds on the flux constraint are hugely simpler to obtain than for the full dynamics, because

- 1) The FC is only a part of the dynamics (much dynamics doesn't need to be considered)
- 2) Considering fluctuations without the need to obey Maxwell's equations is greatly simplifying Simulations with many degrees of freedom respect these bounds.

  In fact, they apparently "find a way" to approach them rather closely

Testing the analytic result with simulations for ITG with adiabatic electrons ( $ITG_{ae}$ )

Recall: flux constraint for the ITG<sub>ae</sub> cannot be satisfied if:  $F_p \ge 0.6$ 

#### Simulation proceedure to test this: start with linear instabilites

- For various TB-like geometries and paramters, FIX a large ion Temperture gradient (R/L $_{\rm T}$   $^{\sim}$  10-80) and increase density gradient (R/L $_{\rm n}$ )
- Well known: with adiabatic electrons, density gradients can't drive an instability, because the density gradient can't be relaxed
  - This is an energetic argument that free energy in density gradients is not accessible with adiabatic electrons

BUT the constant dT/dx is **always** a strong free energy source in the simulations as dn/dx increases

There is no **energetic** reason why the large temperature gradient can't drive instabilitesquit the contrary, strong instabilites would be expected to be present based upon typical behavior in other physical systems

• It is also well known that there can be some linear stability threshold in  $\eta_i = L_n/L_T - WHAT$  fundamantal dynamic is preventing the system with MANY degrees of freedom from accessing the strong T gradient THAT IS ALWAYS PRESENT?

Is it merely a quirk of some linear dispersion relations, or is there a more basic reason?

Yes, the basic reason is the solubility of the FC and because of this origin, there is a striking universality of behavior

### First analytic result: for the ITG with adiabatic electrons ( $ITG_{ae}$ )

Define a normalized density gradient:

$$F_p = \frac{T \ dn/dx}{d \ (nT)/dx} = \frac{Fraction \ of \ the \ density \ gradient}{In \ the \ pressure \ gradient}$$

The flux constraint for the ITG<sub>ae</sub> cannot be satisfied if:

 $F_p \geq 0.6$ 

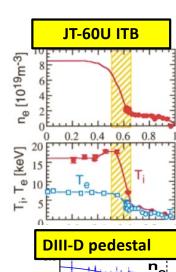
Contrast the simplicity of this criterion to the complexity of a dispersion relation that can't be solved analytically!!

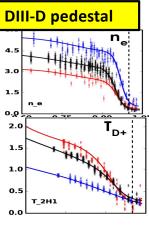
#### Simulation proceedure to test this: we'll start with linear instabilites

- For various TB-like geometries and paramters, FIX a large Temperture gradient (R/L $_{\rm T}$   $^{\sim}$  10-80) and increase density gradient (R/L $_{\rm n}$ )
- With adiabatic electrons, density gradients can't drive an instability, BUT the constant dT/dx is **always** a strong free energy source even as dn/dx increases
- Well known that there can be some linear stability threshold in  $\eta_i = L_n/L_T$  –

But, WHAT fundamental dynamic is preventing the system with MANY degrees of freedom from accessing the strong T gradient THAT IS ALWAYS PRESENT?

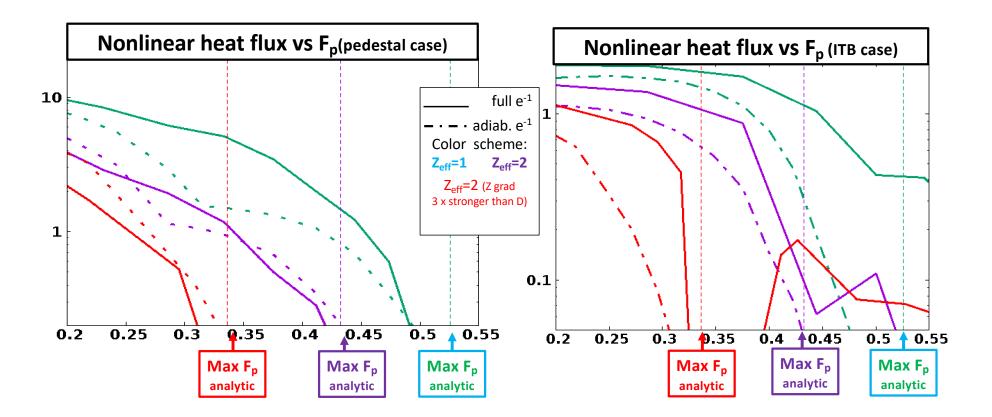
Is it merely a quirk of some linear dispersion relations, or is there a more basic reason?





# Simulations with full electrons and electromagnetic effects for TB cases behave similarly to $ITG_{ae}$ , but the electrons reduce the stabilization

• Sometimes the  $D_{mixing}$  can behave similarly to the ITG<sub>ae</sub> for both pedestal and ITB cases:

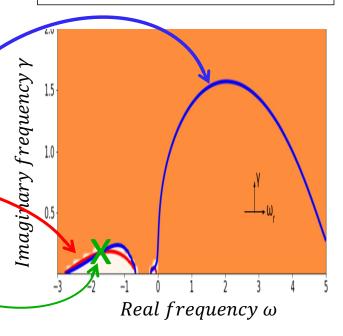


We have developed a technique to reveal dynamics that is much more powerful than a linear dispersion relation

- When non-adiabatic electrons are included, robust or simple analytic results are difficult
- To verify the crucial importance of the constraint, we use SKiM
- Make a plot in the omega upper half plane of:
  - 1) The  $(\omega, \gamma)$  that solve the free energy balance equation
  - 2) The  $(\omega, \gamma)$  that solve the Flux constraint
  - 3) The eigenfrequency is at the intersection

It is evident that the Flux Constraint curve is responsible for forcing the growth rate to be small

 This is true for a very large range of TB parameters when gradients are large, including specific experimental cases A plot, in the upper half plane of  $(\omega,\gamma)$  of the Free-Energy equation and the Flux Constraint for a TB case with weak ITG/TEM



#### Analysis of TB cases with full electrons finds: again, the flux constraint controls stability

- In actual conditions, the modes have important ITG drive and important contributions from electron dynamics
- A mode that is 51% TEM and 49% ITG can hardly be understood as either a TEM mode or an ITG mode- its properties are some mixture of both, and it behavior cannot be relaibly "undertood" from the properties of either
- The Free-Energy balance equation and FC are common to all modes, irrespective of whether they are ITG, TEM, or any combination
- Distinguishing a mode as "mainly ITG" or "mainly TEM" is irrelevant to the analysis
- We regard this as a good thing, since the actual mode cannot realistically be considered as one or the other
- I note: in my interaction with others, many regard the distinct mode types of ITG and TEM as having such a central role in the dynamics that they are aghast at the suggestion that an appropriate understand is either possible or desireable without having to make these distinctions. We disagree.